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UNIVERSIDADE DE CIÊNCIA E TECNOLOGIA DE MACAU
MACAU UNIVERSITY OF SCIENCE AND TECHNOLOGY

澳門四高校聯合入學考試(語言科及數學科)

Joint Admission Examination for Macao Four Higher Education Institutions (Languages and Mathematics)

2023 試題及參考答案
2023 Examination Paper and Suggested Answer

數學正卷 Mathematics Standard Paper

<u>第一部份</u> **選擇題。請選出每題之最佳答案。**

1. 若集合 $M=\{x\mid x^2-2x-8\geq 0\}$，$N=\{x\mid 0<x<6\}$，則 $M\cap N=$ (　　)。

A. $[-2,4]$　　B. $[-2,0)$　　C. $(0,4]$　　D. $(0,6)$　　E. $[4,6)$

2. 若多項式 $f(x)$ 除以 x^2-x-6，餘式為 $3x-2$，則 $f(3)=$ (　　)。

A. -2　　B. 0　　C. 3　　D. 7　　E. 9

3. $\log_9 125\times\log_{12}17\times\log_{25}3\times\log_{17}12=$ (　　)。

A. $\log_{17}3$　　B. $\dfrac{1}{2}$　　C. $\dfrac{3}{4}$　　D. $\log_3 35$　　E. $\log_{17}12$

4. 方程 $x^2-3x+4\sqrt{x^2-3x}=12$ 的解集為 (　　)。

A. $\{-1\}$　　B. $\{2,-6\}$　　C. $\{-1,4\}$　　D. $\{4\}$　　E. $\{3\}$

5. 已知 a 為常數且二次方程 $4a^2x^2+2(a+3)x+9=0$ 只有一個實根，則 $a=$ (　　)。

A. $\dfrac{3}{5}$　　B. -1 或 $\dfrac{3}{2}$　　C. $\dfrac{3}{2}$　　D. $-\dfrac{3}{7}$ 或 $\dfrac{3}{5}$　　E. 任意實數

6. $\left(2\sqrt{x}-\dfrac{1}{\sqrt{x}}\right)^6$ 展開式中的常數項為 (　　)。

A. -8　　B. 8　　C. -160　　D. 160　　E. 1

7. 函數 $f(x)=ax^2+4x+1$（$a\in\mathbb{R}$ 為常數）在區間 $(2,4)$ 上遞增，則 a 的取值範圍為 (　　)。

A. $\left[-\dfrac{1}{2},0\right)$　　B. $\left(0,\dfrac{1}{2}\right]$　　C. $\left[-\dfrac{1}{2},\dfrac{1}{2}\right]$　　D. $\left[-\dfrac{1}{2},\infty\right)$　　E. $\left[\dfrac{1}{2},\infty\right)$

8. 設 $f(x)=\begin{cases}\log_2 x, & 0<x\leq 4\\ x^2-8x+17, & x>4\end{cases}$。不等式 $f(\dfrac{1}{2}-3|x|)+f(5)>0$ 的解為 (　　)。

A. $-\dfrac{1}{12}<x<\dfrac{1}{12}$　　B. $-\dfrac{1}{6}<x<\dfrac{1}{6}$　　C. $-\dfrac{1}{4}<x<\dfrac{1}{4}$

D. $-\dfrac{1}{3}<x<\dfrac{1}{3}$　　E. $-\dfrac{1}{2}<x<\dfrac{1}{2}$

9. 一直立的圓柱形水箱的內半徑為 3 米，高為 8 米，目前水深 5 米。如果將一個半徑為 2 米的球體放入水箱內，且球體完全浸入水中，則水位將上升 (　　) 米。

A. $\frac{2}{3}$　　B. $\frac{3}{2}$　　C. 1　　D. $\frac{16}{27}$　　E. $\frac{32}{27}$

10. 在等差數列中，第 7 項是 80 及第 16 項是 26，則第 34 項為 (　　)。

A. -6　　B. -82　　C. -88　　D. -198　　E. -204

11. 已知點 $A(3,-8)$ 和 $B(-7,4)$。通過 AB 的中點並且垂直於 $3x-4y+14=0$ 的直線方程為 (　　)。

A. $4x+3y+14=0$　　B. $3x+4y+14=0$　　C. $3x-4y-14=0$

D. $4x-3y+14=0$　　E. $4x+3y-14=0$

12. 雙曲線 $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1\ (a,b>0)$ 的離心率是 3，則 $\frac{b^2+2}{a}$ 的最小值為 (　　)。

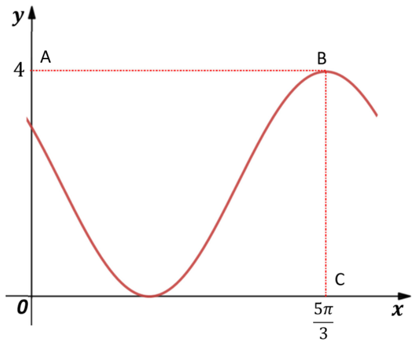
A. 2　　B. $2\sqrt{2}$　　C. $2\sqrt{3}$　　D. 4　　E. 8

13. 設 A 和 B 是第二象限中的角，且 $\sin A=\frac{2}{5}$ 及 $\sin B=\frac{4}{5}$，則 $\sin(A+B)=$ (　　)。

A. $\frac{-6-4\sqrt{21}}{25}$　　B. $\frac{13}{25}$　　C. $\frac{18}{25}$

D. $\frac{-12-2\sqrt{21}}{25}$　　E. $\frac{12+2\sqrt{21}}{25}$

14. 右圖所示為函數 $y=a\sin(x-\frac{\pi}{6})+b$ 的圖像，其中 a 和 b 為常數，則 (　　)。



A. $a=-4$ 及 $b=4$　　B. $a=-2$ 及 $b=2$

C. $a=2$ 及 $b=-2$　　D. $a=4$ 及 $b=-4$

E. 以上皆非

15. 點 $A(-2,3)$ 繞原點 O 順時針方向旋轉 $90°$ 到點 B。點 C 與點 B 關於 x 軸對稱。點 C 向下平移三個單位到點 D，則 D 點坐標為 (　　)。

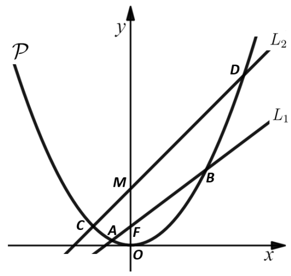
A. $(-3,-1)$　　B. $(-3,0)$　　C. $(-4,0)$　　D. $(2,0)$　　E. $(3,-5)$

<u>第二部份</u> 解答題。

1. 有一枚不均勻的硬幣，其正面朝上的概率是 $\frac{1}{4}$。

   (a) 連續十次投擲此硬幣，求獲得最多一次正面朝上的概率。 (3 分)
   (b) 求在第十次投擲才取第一次獲得正面朝上的概率。 (2 分)
   (c) 求在第十次投擲取得第三次獲得正面朝上的概率。 (3 分)

2. 在右圖中，拋物線 $\mathcal{P}$: $x^2 = 4y$ 的焦點為 F。經過焦點 F 斜率為 $\frac{3}{4}$ 的直線 L_1 與拋物線 $\mathcal{P}$ 的交點為 A 和 B。另外一條斜率為 1 的直線 L_2 與拋物線 $\mathcal{P}$ 的交點為 C 和 D，與 y 軸的交點為 M。



   (a) 求焦點 F 的坐標。 (2 分)
   (b) 求線段 AB 的長度。 (3 分)
   (c) 若 $|DM| = 3|CM|$，求線段 CD 的長度。 (3 分)

3. 已知 $S_n = 3^{n+1} - 2k$ 是等比數列 $\{a_n\}_{n\geq 1}$ 的前 n 項和，這裡 $k \in \mathbb{R}$ 為常數。

   (a) 求 k 及 a_n。 (3 分)
   (b) 設 $b_n = \frac{1}{a_n} + \log_2 a_n$，求 b_n 的前 n 項和 T_n。 (3 分)
   (c) 設 $c_n = \frac{2}{a_n}$，求 $f(n) = -5c_n^2 + c_n$ 取得最大值時 n 的值。 (2 分)

4. 已知函數 $f(x) = \sqrt{3}\sin(2wx) - 2\cos^2(wx)$ 的最小正週期為 3π。

   (a) 求 $f(x)$ 的表達式。 (4 分)
   (b) 在 $\triangle ABC$ 中，若 $f(C) = 0$，且 $2\sin^2 B = \cos B + \cos(A - C)$，求 $\sin A$ 的值。 (4 分)

5. 設 x, y 滿足 $\begin{cases} 3x + 2y - 13 \geq 0 \\ x \leq 5 \\ 2x - 2y + 3 \geq 0 \end{cases}$。

   (a) 畫出滿足以上不等式組的區域。 (2 分)
   (b) 設 $z = \frac{y}{x}$，求 z 的取值範圍。 (3 分)
   (c) 設 $t = x^2 + y^2$，求 t 的最小值。 (3 分)

參考答案

<u>第一部份</u> 選擇題。

題目編號	最佳答案
1	E
2	D
3	C
4	C
5	D
6	C
7	D
8	A
9	E
10	B
11	A
12	E
13	A
14	B
15	E

第二部份 解答題。

1. (a) 連續投擲十次硬幣，十次都是反面向上的概率是 $(1-\frac{1}{4})^{10}=(\frac{3}{4})^{10}$。連續投擲十次硬幣，剛好出現一次正面向上的概率是 ${}_{10}C_1\frac{1}{4}(1-\frac{1}{4})^{10-1}=\frac{5}{2}(\frac{3}{4})^9$。因此，最多一次正面向上的概率是 $(\frac{3}{4})^{10}+\frac{5}{2}(\frac{3}{4})^9=\frac{13}{4}\times(\frac{3}{4})^9$。

   (b) 前九次投擲都是反面向上且第十次投擲是正面向上的概率是 $(1-\frac{1}{4})^{10-1}\times\frac{1}{4}=\frac{3^9}{4^{10}}$。

   (c) 前九次投擲中剛好出現兩次正面向上的概率是 ${}_9C_2(\frac{1}{4})^2(1-\frac{1}{4})^{9-2}$。因此，在第十次投擲取得第三次正面向上的概率是 ${}_9C_2(\frac{1}{4})^2(1-\frac{1}{4})^{9-2}\times\frac{1}{4}=(\frac{3}{4})^9$。

2. (a) 焦點 F 的坐標為 $(0,1)$。

   (b) 直線 L_1 的方程為 $y=\frac{3}{4}x+1$，聯立直線 L_1 和拋物線 $\mathcal{P}$ 的方程並消去變量 y 得 $x^2=3x+4$，求解得 $x=-1$ 或 4。點 A 和點 B 的坐標分別為 $A(-1,\frac{1}{4})$ 和 $B(4,4)$。於是，我們得到 $|AB|=\frac{25}{4}$。

   (c) 設直線 L_2 的方程為 $y=x+t$，設點 C 和點 D 的橫坐標分別為 x_1 和 x_2。聯立直線 L_2 和拋物線 $\mathcal{P}$ 的方程並消去變量 y 得 $x^2-4x-4t=0$。由韋達定理得 $x_1+x_2=4$。又因為 $|DM|=3|CM|$，所以 $x_2=-3x_1$。進一步，求得 $x_1=-2$ 和 $x_2=6$。因此 $|CD|=\sqrt{1^2+1}|x_2-x_1|=8\sqrt{2}$。

3. (a) 由題意得到 $a_1=9-2k$，$a_2=S_2-S_1=27-9=18$ 及 $a_3=S_3-S_2=54$。因為 $\{a_n\}_{n\geq1}$ 為等比數列，於是 $a_1a_3=a_2^2$，從而求得首項 $a_1=6$，公比 $q=3$ 以及 $k=3/2$。因此 $a_n=a_1\times q^{n-1}=2\times3^n$。

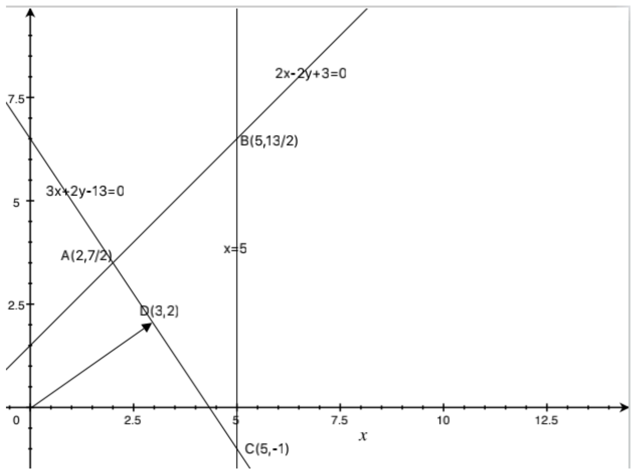
   (b) 因為 $b_n=\frac{1}{2\times3^n}+\log_2(2\times3^n)=\frac{1}{2\times3^n}+1+n\log_2 3$，所以 $T_n=\frac{1}{4}\left(1-\frac{1}{3^n}\right)+n+\frac{n(n+1)}{2}\log_2 3$。

   (c) 因為 $c_n=3^{-n}$，所以 $f(n)=-5(3^{-n})^2+3^{-n}=-5(\frac{1}{3^n}-\frac{1}{10})^2+\frac{1}{20}$。因此當 $n=2$ 時，函數 $f(n)$ 取得最大值 $\frac{4}{81}$。

4. (a) 由二倍角公式可得 $f(x)=\sqrt{3}\sin 2wx-(1+\cos 2wx)$，於是 $f(x)=2(\frac{\sqrt{3}}{2}\sin 2wx-\frac{1}{2}\cos 2wx)-1=2\sin(2wx-\theta)-1$，其中 $\sin\theta=\frac{1}{2}$ 以及 $\cos\theta=\frac{\sqrt{3}}{2}$。因此，$\theta=\frac{\pi}{6}+2k\pi$。因為 $f(x)$ 的最小正週期為 $\frac{2\pi}{2w}=3\pi$，所以 $2w=\frac{2}{3}$，從而 $f(x)=2\sin(\frac{2}{3}x-\frac{\pi}{6})-1$。

(b) 由 $f(C) = 2\sin(\frac{2}{3}C - \frac{\pi}{6}) - 1 = 0$ 得 $\sin(\frac{2}{3}C - \frac{\pi}{6}) = \frac{1}{2}$。因為 $-\frac{\pi}{6} < \frac{2}{3}C - \frac{\pi}{6} < \frac{\pi}{2}$，所以 $\frac{2}{3}C - \frac{\pi}{6} = \frac{\pi}{6}$，從而 $C = \frac{\pi}{2}$ 及 $A + B = \frac{\pi}{2}$。又因為 $2\sin^2 B = \cos B + \cos(A - \frac{\pi}{2})$，所以 $2\sin^2 B = \sin A + \sin A$，即 $\sin^2 B = \sin A$，從而有 $1 - \sin^2 A = \sin A$。因此 $\sin A = \frac{\sqrt{5}-1}{2}$。

5. (a)



(b) y/x 為連結點 $P(x, y)$ 和原點的直線的斜率。求解三條直線的交點可得 $A(2, 7/2)$,$B(5, 13/2)$ 和 $C(5, -1)$。那麼當點 P 在給定區域內變動時，z 的最小值可以在點 C 取得，最大值可在點 A 處取得，因此 $-1/5 \le z \le 7/4$。

(c) t 為區域中的點 $P(x, y)$ 和原點間距離的平方。距離原點最近的點 D 在直線 $3x + 2y - 13 = 0$ 上，並且 OD 垂直於該直線。因此，直線 OD 的斜率為 $2/3$，方程為 $y = 2x/3$。直線 OD 與直線 $3x + 2y - 13 = 0$ 的交點為 $(x, y) = (3, 2)$，因此 t 在 $(x, y) = (3, 2)$ 取得最小值 13。

<u>Part I</u> Multiple choice questions. Choose the *best answer* for each question.

1. Let $M = \{x \mid x^2 - 2x - 8 \geq 0\}$ and $N = \{x \mid 0 < x < 6\}$, then $M \cap N =$ ( ).

   A. $[-2, 4]$ B. $[-2, 0)$ C. $(0, 4]$ D. $(0, 6)$ E. $[4, 6)$

2. If we divide the polynomial $f(x)$ by $x^2 - x - 6$ and the remainder is $3x - 2$, then $f(3) =$ ( ).

   A. -2 B. 0 C. 3 D. 7 E. 9

3. $\log_9 125 \times \log_{12} 17 \times \log_{25} 3 \times \log_{17} 12 =$ ( ).

   A. $\log_{17} 3$ B. $\frac{1}{2}$ C. $\frac{3}{4}$ D. $\log_3 35$ E. $\log_{17} 12$

4. The set of solutions for the equation $x^2 - 3x + 4\sqrt{x^2 - 3x} = 12$ is ( ).

   A. $\{-1\}$ B. $\{2, -6\}$ C. $\{-1, 4\}$ D. $\{4\}$ E. $\{3\}$

5. Let a be a constant and suppose the quadratic equation $4a^2x^2 + 2(a + 3)x + 9 = 0$ has exactly one real solution. Then $a =$ ( ).

   A. $\frac{3}{5}$ B. -1 or $\frac{3}{2}$ C. $\frac{3}{2}$
   D. $-\frac{3}{7}$ or $\frac{3}{5}$ E. any real number

6. The constant term in the expansion of $\left(2\sqrt{x} - \frac{1}{\sqrt{x}}\right)^6$ is ( ).

   A. -8 B. 8 C. -160 D. 160 E. 1

7. The function $f(x) = ax^2 + 4x + 1$ ($a \in \mathbb{R}$ is a constant) is increasing on the open interval $(2, 4)$. Then the range of a is ( ).

   A. $\left[-\frac{1}{2}, 0\right)$ B. $\left(0, \frac{1}{2}\right]$ C. $\left[-\frac{1}{2}, \frac{1}{2}\right]$ D. $\left[-\frac{1}{2}, \infty\right)$ E. $\left[\frac{1}{2}, \infty\right)$

8. Let $f(x) = \begin{cases} \log_2 x, & 0 < x \leq 4 \\ x^2 - 8x + 17, & x > 4 \end{cases}$. The solution of the inequality $f(\frac{1}{2} - 3|x|) + f(5) > 0$ is ( ).

   A. $-\frac{1}{12} < x < \frac{1}{12}$ B. $-\frac{1}{6} < x < \frac{1}{6}$ C. $-\frac{1}{4} < x < \frac{1}{4}$

   D. $-\frac{1}{3} < x < \frac{1}{3}$ E. $-\frac{1}{2} < x < \frac{1}{2}$

9. An upright cylindrical water tank has an inner radius of 3 meters and a height of 8 meters, and the current water depth is 5 meters. If a sphere with a radius of 2 meters is placed into the water tank and the sphere is completely immersed in the water, the water level rises by (　　) meters.

A. $\frac{2}{3}$　　B. $\frac{3}{2}$　　C. 1　　D. $\frac{16}{27}$　　E. $\frac{32}{27}$

10. In an arithmetic sequence, the 7th term is 80 and the 16th term is 26. Then the 34th term is (　　).

A. -6　　B. -82　　C. -88　　D. -198　　E. -204

11. Let A and B be the points $(3, -8)$ and $(-7, 4)$ respectively. An equation of the line passing through the midpoint of AB and perpendicular to $3x - 4y + 14 = 0$ is (　　).

A. $4x + 3y + 14 = 0$　　B. $3x + 4y + 14 = 0$　　C. $3x - 4y - 14 = 0$

D. $4x - 3y + 14 = 0$　　E. $4x + 3y - 14 = 0$

12. If the eccentricity of the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ $(a, b > 0)$ is 3, then the minimum value of $\frac{b^2+2}{a}$ is (　　).

A. 2　　B. $2\sqrt{2}$　　C. $2\sqrt{3}$　　D. 4　　E. 8

13. Let A and B be angles in the second quadrant such that $\sin A = \frac{2}{5}$ and $\sin B = \frac{4}{5}$. Then $\sin(A + B) =$ (　　).

A. $\frac{-6-4\sqrt{21}}{25}$　　B. $\frac{13}{25}$　　C. $\frac{18}{25}$

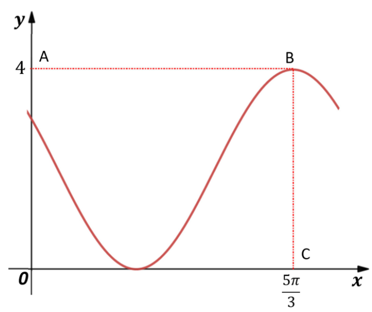
D. $\frac{-12-2\sqrt{21}}{25}$　　E. $\frac{12+2\sqrt{21}}{25}$

14. The right figure shows the graph of $y = a\sin(x - \frac{\pi}{6}) + b$, where a and b are constants. Then (　　).

A. $a = -4$ and $b = 4$　　B. $a = -2$ and $b = 2$

C. $a = 2$ and $b = -2$　　D. $a = 4$ and $b = -4$

E. none of the above



15. Point $A(-2, 3)$ is rotated $90°$ clockwise about the origin O to get point B. Points C and B are symmetrical about the x-axis. Point C is translated downward three units to get point D. Then the coordinates of point D are (　　).

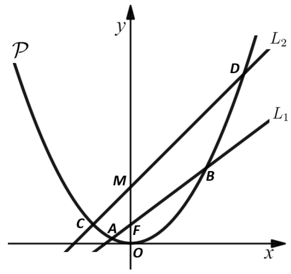
A. $(-3, -1)$　　B. $(-3, 0)$　　C. $(-4, 0)$　　D. $(2, 0)$　　E. $(3, -5)$

<u>Part II</u> Problem-solving questions.

1. A coin is unfair that the probability of a head facing up is $\frac{1}{4}$.

   (a) Find the probability of obtaining at most one head facing up in 10 successive tosses. (3 marks)
   (b) Find the probability that the 10th toss will be the first of obtaining head facing up. (2 marks)
   (c) Find the probability that the 10th toss will be the third of obtaining head facing up. (3 marks)

2. In the right figure, the parabola $\mathcal{P}: x^2 = 4y$ has its focus F. The straight line L_1 of slope $\frac{3}{4}$ passing through the focus F intersects the parabola $\mathcal{P}$ at points A and B. Another straight line L_2 of slope 1 intersects the parabola $\mathcal{P}$ at points C and D, and intersects the y-axis at point M.



   (a) Find the coordinates of the focus F. (2 marks)
   (b) Find the length of segment AB. (3 marks)
   (c) If $|DM| = 3|CM|$, find the length of segment CD. (3 marks)

3. Let $S_n = 3^{n+1} - 2k$ be the nth sum of the geometric sequence $\{a_n\}_{n\geq 1}$. Here $k \in \mathbb{R}$ is a constant.

   (a) Find k and a_n. (3 marks)
   (b) Let $b_n = \frac{1}{a_n} + \log_2 a_n$. Find the sum T_n of the first n terms for the sequence b_n. (3 marks)
   (c) Let $c_n = \frac{2}{a_n}$. Find n where $f(n) = -5c_n^2 + c_n$ obtains its maximum value. (2 marks)

4. The minimal positive period of the function $f(x) = \sqrt{3}\sin(2wx) - 2\cos^2(wx)$ is 3π.

   (a) Find the expression of $f(x)$. (4 marks)
   (b) In $\triangle ABC$, if $f(C) = 0$ and $2\sin^2 B = \cos B + \cos(A - C)$, find the value of $\sin A$. (4 marks)

5. Let x, y satisfy $\begin{cases} 3x + 2y - 13 \geq 0 \\ x \leq 5 \\ 2x - 2y + 3 \geq 0 \end{cases}$.

   (a) Sketch the region satisfying the above system of inequalities. (2 marks)
   (b) Let $z = \frac{y}{x}$. Find the range of z. (3 marks)
   (c) Let $t = x^2 + y^2$. Find the minimum value of t. (3 marks)

Suggested Answer

<u>Part I</u> Multiple choice questions.

Question Number	Best Answer
1	E
2	D
3	C
4	C
5	D
6	C
7	D
8	A
9	E
10	B
11	A
12	E
13	A
14	B
15	E

<u>Part II</u> Problem-solving questions.

1. (a) If a coin is tossed 10 times consecutively, the probability of getting tails 10 times is $(1-\frac{1}{4})^{10} = (\frac{3}{4})^{10}$. The probability of getting exactly one head in ten consecutive tosses of a coin is ${}_{10}C_1\frac{1}{4}(1-\frac{1}{4})^{10-1} = \frac{5}{2}(\frac{3}{4})^9$. Therefore, the probability of at most one head up is $(\frac{3}{4})^{10} + \frac{5}{2}(\frac{3}{4})^9 = \frac{13}{4} \times (\frac{3}{4})^9$.

   (b) The probability that the first 9 tosses are tails and the 10th toss is head is $(1-\frac{1}{4})^{10-1} \times \frac{1}{4} = \frac{3^9}{4^{10}}$.

   (c) The probability of getting exactly two heads in the first 9 tosses is ${}_9C_2(\frac{1}{4})^2(1-\frac{1}{4})^{9-2}$. Therefore, the probability of getting a third head on the tenth toss is ${}_9C_2(\frac{1}{4})^2(1-\frac{1}{4})^{9-2} \times \frac{1}{4} = (\frac{3}{4})^9$.

2. (a) The coordinates of the focus F is $(0,1)$.

   (b) The equation of the straight line L_1 is $y = \frac{3}{4}x + 1$. Combining the equations of the straight line L_1 and the parabola $\mathcal{P}$, we can get $x^2 = 3x + 4$. Solving the quadratic equation, one has $x = -1$ or 4. Then the coordinates of points A and B are $A(-1, \frac{1}{4})$ and $B(4,4)$, respectively. Then according to the definition of parabolas, we have $|AB| = \frac{25}{4}$.

   (c) Suppose the equation of the straight line L_2 is $y = x + t$. Suppose the x-coordinates of points C and D are x_1 and x_2, respectively. Combining the equations of the straight line L_2 and the parabola $\mathcal{P}$, we can get $x^2 - 4x - 4t = 0$. Using Weda's Theorem, $x_1 + x_2 = 4$. Since $|DM| = 3|CM|$, we have $x_2 = -3x_1$. Furthermore, we can get $x_1 = -2, x_2 = 6$. Thus, $|CD| = \sqrt{1^2+1}|x_4 - x_3| = 8\sqrt{2}$.

3. (a) From the question, we get $a_1 = S_1 = 9 - 2k, a_2 = S_2 - S_1 = 27 - 9 = 18$ and $a_3 = S_3 - S_2 = 54$. Since $\{a_n\}_{n\geq 1}$ is a geometric sequence, we have $a_1a_3 = a_2^2$, which implies that the first term $a_1 = 6$, the common ratio $q = 3$ and $k = 3/2$. Then we can get $a_n = a_1 \times q^{n-1} = 2 \times 3^n$.

   (b) Since $b_n = \frac{1}{2\times 3^n} + \log_2(2\times 3^n) = \frac{1}{2\times 3^n} + 1 + n\log_2 3$, $T_n = \frac{1}{4}\left(1 - \frac{1}{3^n}\right) + n + \frac{n(n+1)}{2}\log_2 3$.

   (c) Since $c_n = 3^{-n}$, $f(n) = -5(3^{-n})^2 + 3^{-n} = -5(\frac{1}{3^n} - \frac{1}{10})^2 + \frac{1}{20}$. When $n = 2$, the function $f(n)$ obtains its maximum value $\frac{4}{81}$.

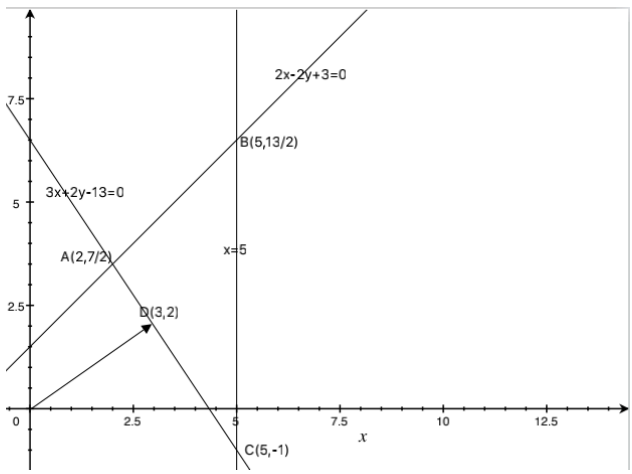
4. (a) By the double-angle formula, $f(x) = \sqrt{3}\sin 2wx - (1 + \cos 2wx)$. So $f(x) = 2(\frac{\sqrt{3}}{2}\sin 2wx -$

$\frac{1}{2}\cos 2wx) - 1 = 2\sin(2wx - \theta) - 1$, with $\sin\theta = \frac{1}{2}$ and $\cos\theta = \frac{\sqrt{3}}{2}$. Thus $\theta = \frac{\pi}{6} + 2k\pi$. Since the least period of $f(x)$ is $\frac{2\pi}{2w} = 3\pi$, we get $2w = \frac{2}{3}$ and can write $f(x) = 2\sin(\frac{2}{3}x - \frac{\pi}{6}) - 1$.

(b) Since $f(C) = 2\sin(\frac{2}{3}C - \frac{\pi}{6}) - 1 = 0$, we have $\sin(\frac{2}{3}C - \frac{\pi}{6}) = \frac{1}{2}$. By observing that $-\frac{\pi}{6} < \frac{2}{3}C - \frac{\pi}{6} < \frac{\pi}{2}$, we can get $\frac{2}{3}C - \frac{\pi}{6} = \frac{\pi}{6}$. Then $C = \frac{\pi}{2}$ and $A + B = \frac{\pi}{2}$. We have $2\sin^2 B = \cos B + \cos(A - \frac{\pi}{2})$ which implies that $2\sin^2 B = \sin A + \sin A$. Then $1 - \sin^2 A = \sin A$, and $\sin A = \frac{\sqrt{5}-1}{2}$.

5. Answer:

(a)



(b) y/x is the slope of straight line jointing point $P(x, y)$ and the origin. Intersections of the given straight lines are $A(2, 7/2)$, $B(5, 13/2)$ and $C(5, -1)$. When point P varies inside the given region, the minimum value of z can be obtained at point C and the maximum value can be obtained at point A. Then$-1/5 \le z \le 7/4$.

(c) t is the square of the distance between point $P(x, y)$ in the given region and the origin. The nearest point to the origin is the point D lying on the line $3x + 2y - 13 = 0$ and OD is perpendicular to this straight line. The slope of the line OD should be $2/3$ and the equation is $y = 2x/3$. Thus, the intersection of lines OD and $3x + 2y - 13 = 0$ is $(x, y) = (3, 2)$. Therefore, t obtains its minimum value 13 at point $(x, y) = (3, 2)$.